Name Solutions

EE 311 Final Exam Spring 2013

May 2, 2013

Closed Text and Notes, No calculators

1) Be sure you have 17 pages and the additional pages of equations.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

5) This exam is worth 150 points.

(5 pts) 1. What geometry is described by $\theta = \pi$? Choose the most specific

A) xy plane

B) cone

C) line

D) –z axis

(5 pts). 2. The intersection of the surfaces r = 1m and $\theta = \frac{\pi}{2}$ is

A) the xy plane

B) a cone

C) a line

D) a circle

(6 pts) 3. Convert the point (1, 1, 1) in Cartesian coordinates to cylindrical coordinates

$\rho = \sqrt{x^2+y^2} = \sqrt{2}$

$\phi = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$ radians

$z = 1$

$(\sqrt{2}, \frac{\pi}{4}, 1)$

(5 pts) 4. A -3 C charge is place at the location (1 m, 3 m, -2 m) and it experiences a force of $\mathbf{F}=(21\hat{\mathbf{a}}_x - 6\hat{\mathbf{a}}_y + 12\hat{\mathbf{a}}_z)\mathrm{N}$. What is the electric field intensity at (1 m, 3 m, -2 m)?

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{(21\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)\,N}{-3\,C}$$

$$= (-7\hat{a}_x + 2\hat{a}_y - 4\hat{a}_z)\,\frac{N}{C}$$

$$= (-7\hat{a}_x + 2\hat{a}_y - 4\hat{a}_z)\,\frac{V}{m}$$

(8 pts) 5 Fill in the table with the standard units for the following

Magnetic flux density, **B**	$\frac{Wb}{m^2} = T$
Magnetic field intensity, **H**	$\frac{A}{m}$
Electric Field Intensity, **E**	$\frac{V}{m}$
Electric Flux Density, **D**	$\frac{C}{m^2}$
Polarization, **P**	$\frac{C}{m^2}$
Magnetization, **M**	$\frac{A}{m}$
Electric flux, **Ψ**	C
Magnetic flux, **Ψ**	Wb

(8 pts) 6. For r < 1m the permittivity is $2\varepsilon_0$, For r > 1m the permittivity is $4\varepsilon_0$. On the interface between the dielectrics, at r = 1m, there is a surface charge density of $\rho_s = 1\frac{C}{m^2}$ and for r > 1 m there are no free charges. If $\mathbf{D} = \frac{10}{r^2}\frac{C}{m^2}\,\hat{\mathbf{a}}_r$ for r < 1m, what is D for r > 1m.

$\vec{D} = \frac{10}{r^2}\frac{C}{m^2}\hat{a}_r$ describes an electric flux density caused by a point charge at the origin. A dielectric does not affect a $\vec{D}$ field. Find the $\vec{D}$ field caused by the charge on the surface of the sphere and use superposition to get the $\vec{D}$ field for $r > 1m$

$$\oint \vec{D} \cdot \vec{ds} = Q_{encl}$$

$$D\, 4\pi r^2 = \left(1\,\frac{C}{m^2}\right) 4\pi (1m)^3 = 4\pi\ C$$

$$\vec{D} = \frac{1}{r^2}\hat{a}_r \quad \text{for } r > 1m$$

$$\vec{D} = \left(\frac{10}{r^2} + \frac{1}{r^2}\right)\frac{C}{m^2}\hat{a}_r \quad \text{for } r > 1m$$

$$\vec{D} = \frac{11}{r^2}\frac{C}{m^2}\hat{a}_r \quad \text{for } r > 1m$$

(10 pts) 7. A conducting metal sphere has radius 1 m and is centered at the origin. A total charge of $\frac{10^{-9}}{36}$C is on the sphere and $V(\infty) = 5V$. Note $\varepsilon_o = \frac{10^{-9}}{36\pi}\frac{F}{m}$

A) What is $V(r=2m, \theta=\frac{\pi}{4}, \phi=\frac{\pi}{4}) - V(r=3m, \theta=\frac{\pi}{3}, \phi=\frac{\pi}{4})$?

First find the electric field intensity

$\oint \vec{D}\cdot d\vec{s} = Q_{encl}$ for $r > 1m$ $\quad D4\pi r^2 = \frac{10^{-9}}{36}C$

$$\vec{D} = \frac{(10^{-9}/36)C}{4\pi r^2}\hat{a}_r \quad \text{for } r > 1m$$

$$= 0 \quad \text{for } r < 1m$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_o} = \frac{1}{4r^2}\hat{a}_r \frac{V}{m} \quad \text{for } r > 1m$$

$$= 0 \quad \text{for } r < 1m$$

$$V(2m) - V(3m) = -\int_{3m}^{2m} \frac{1}{4r^2}\hat{a}_r \cdot dr\hat{a}_r = -\int_{3m}^{2m}\frac{dr}{4r^2} = \frac{1}{4r}\Big|_3^2 V$$

$$= \left(\frac{1}{8} - \frac{1}{12}\right)V$$

$$= \frac{1}{24}V$$

B) What is $V(r=0m, \theta=\frac{\pi}{4}, \phi=\frac{\pi}{4}) - V(r=1m, \theta=\frac{\pi}{3}, \phi=\frac{\pi}{4})$?

Since $E = 0$ for $r < 1m$,

$V(r=0m) = V(r=1m)$

so

$$V(0, \tfrac{\pi}{4}, \tfrac{\pi}{4}) - V(1, \tfrac{\pi}{3}, \tfrac{\pi}{4}) = 0$$

(5 pts) 8 A -5 C and a -2 C charge are 1 m apart. How much energy is stored in this two-charge configuration? Note $\varepsilon_0 = \frac{10^{-9}}{36\pi}\frac{F}{m}$

The amount of energy stored in the charge configuration is the amount of work it took to assemble the charges.

Bringing in the -5C charge and placing it at the origin requires no work. The potential field caused by the -5C charge is $V(r) = \frac{-5}{4\pi\varepsilon_0 r}$ V

The work to position the -2C charge 1m from the -5C charge is

$$\text{energy stored} = W = V(1m)\,Q = \frac{(-5)(-2)}{4\pi\varepsilon_0(1)}\text{ J} = \frac{10}{4\pi\left(\frac{10^{-9}}{36\pi}\right)}\text{ J}$$

$$= 9\times10^{10}\text{ J}$$

(5 pts) 9. A copper rod is 1 m long and has a cross-sectional area of 0.01m^2. There are about $n = 10^{29}\frac{\text{conduction electrons}}{\text{m}^3}$ in copper. A voltage is applied to the ends of the rod so that a current of 1.6 A flows along the 1 m length. What is the average drift velocity of the conduction electrons?

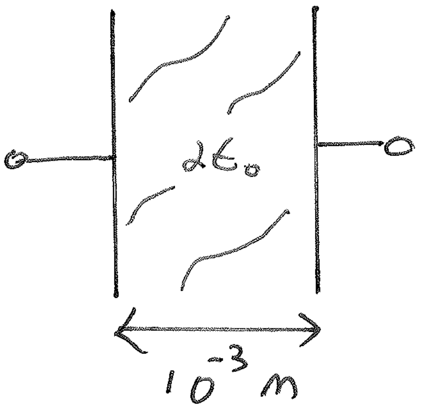
$$I = JA = qnuA = \left(1.6\times10^{-19}\frac{C}{elec}\right)\left(10^{29}\frac{elec}{m^3}\right)u\left(0.01\,m^2\right)$$

$$1.6\text{ A} = \left(1.6\times10^{8}\frac{C}{m}\right)u$$

$$u = \frac{1.6\text{ A}}{1.6\times10^{8}\frac{C}{m}} = \frac{1.6\frac{C}{s}}{1.6\times10^{8}\frac{C}{m}}$$

$$u = 10^{-8}\frac{m}{s}$$

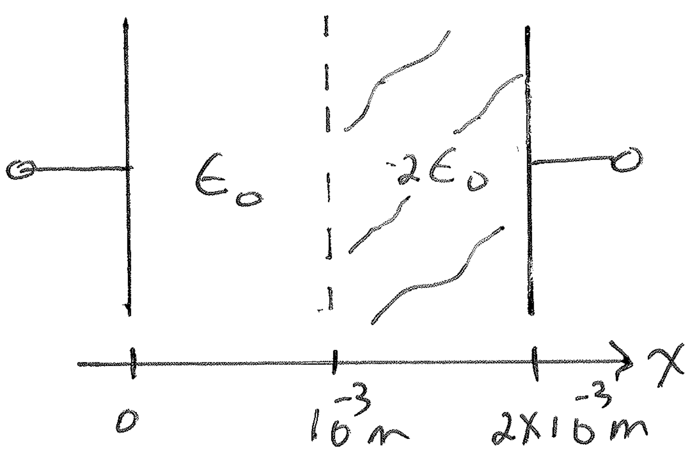
(10 pts) 10. A parallel plate capacitor has plate area 0.01m^2, a plate separation of 1 mm, and a dielectric with dielectric constant $\varepsilon_r = 2$ between the plates. A 10 V battery is connected to the capacitor and then removed. Without disturbing the charge on the plates, the plate separation is increased to 2 mm. What is the potential drop across the plates? Note that half the region between the plates is the original dielectric and half is now free space.



$$E = \frac{10V}{10^{-3}m} = 10^4 \frac{V}{m}$$

$$D = 2\epsilon_0 E = 2\epsilon_0 \times 10^4 \frac{C}{m^2}$$

now pull the plates apart to 2×10^{-3} m



$$D = 2\epsilon_0 \times 10^4 \frac{C}{m^2}, \quad 0 < x < 2\times10^{-3}m$$

$$E = \frac{D}{\epsilon_r \epsilon_0}$$

$$E = 2\times10^4 \frac{V}{m}, \quad 0 < x < 10^{-3}m$$

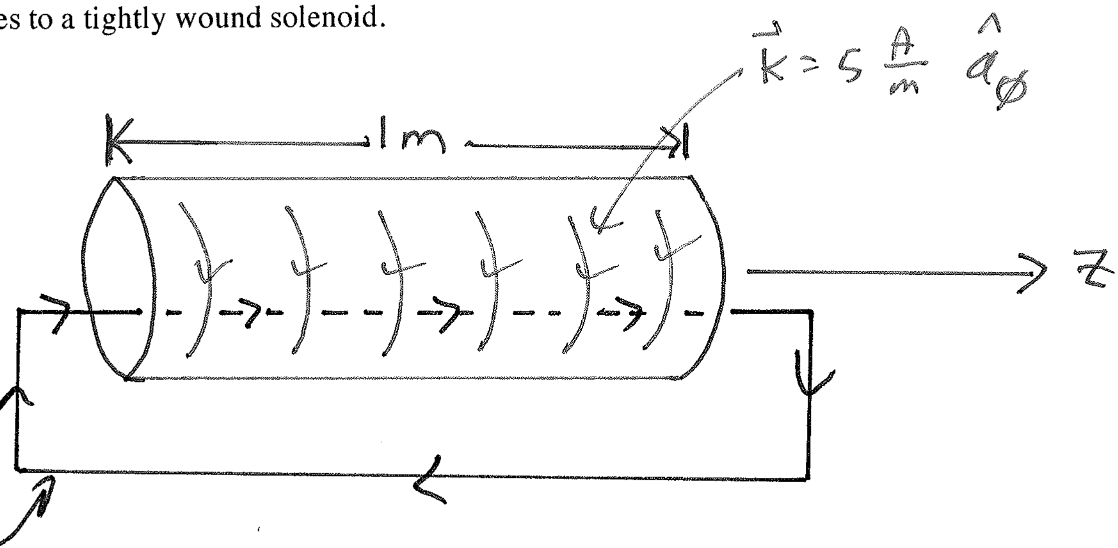
$$= 10^4 \frac{V}{m}, \quad 10^{-3}m < x < 2\times10^{-3}m$$

$$V = -\int_{2\times10^{-3}m}^{0} E\,dx = -\int_{2\times10^{-3}m}^{10^{-3}m} 10^4 \frac{V}{m}\,dx - \int_{10^{-3}m}^{0} 2\times10^4 \frac{V}{m}\,dx$$

$$= \left(10^4 \frac{V}{m}\right)10^{-3}m + \left(2\times10^4 \frac{V}{m}\right)\left(10^{-3}m\right)$$

$$= 10V + 20V = 30V$$

(10 pts) 11. A current of density $\mathbf{K}=5\frac{\text{A}}{\text{m}}\hat{a}_\phi$ is flowing on the surface of a hollow conductor of radius $\rho=1\text{cm}$. (So the current is zero everywhere except at $\rho=1\text{cm}$.) The cylinder is of length 1 m in the z-direction. What is the magnetic field intensity everywhere? Hint think of how this compares to a tightly wound solenoid.



path of integration

outside the cylinder the field is weak so is a negligible contribution to $\oint \vec{H}\cdot d\vec{l}$.

$$\oint H\cdot dl = I_{enclosed}$$

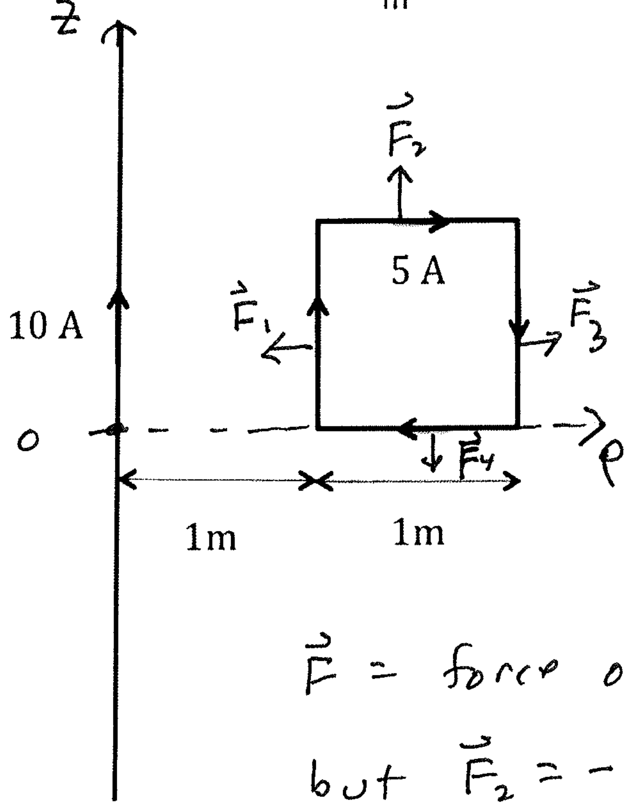
$$H\,(1\text{m}) = \left(5\,\frac{A}{m}\right)(1\text{m})$$

$$\vec{H} = 5\,\frac{A}{m}\,\hat{a}_z \quad \text{inside the cylinder}$$

$$\approx 0 \quad \text{elsewhere}$$

(10 npts) 12. Find the force on the 1m x 1m loop shown. The loop and infinite wire are in the same plane.

Note $\mu_o = 4\pi x 10^{-7} \frac{H}{m}$



First find the magnetic flux density, $\vec{B}$, due to the 10A infinite wire.

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi = \frac{10}{2\pi\rho}\hat{a}_\phi$$

$$\vec{B} = \mu_o \vec{H} = (4\pi \times 10^{-7})\frac{5}{\pi\rho}\hat{a}_\phi$$

$$= \frac{2\times 10^{-6}}{\rho}\hat{a}_\phi \, T$$

$\vec{F}$ = force on the loop $= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

but $\vec{F}_2 = -\vec{F}_4$ so $\vec{F} = \vec{F}_1 + \vec{F}_3$

$$\vec{F}_1 = \int_0^{1m} I\,\vec{dl} \times \vec{B}$$

$$= \int_0^{1m} (5A)\,dz\,\hat{a}_z \times \frac{2\times 10^{-6}}{\rho} T\hat{a}_\phi$$

$$= \int_0^{1} (5A)\frac{2\times 10^{-6}}{1} T\,dz(-\hat{a}_\rho)$$

$$= -10^{-5}\hat{a}_\rho \, N$$

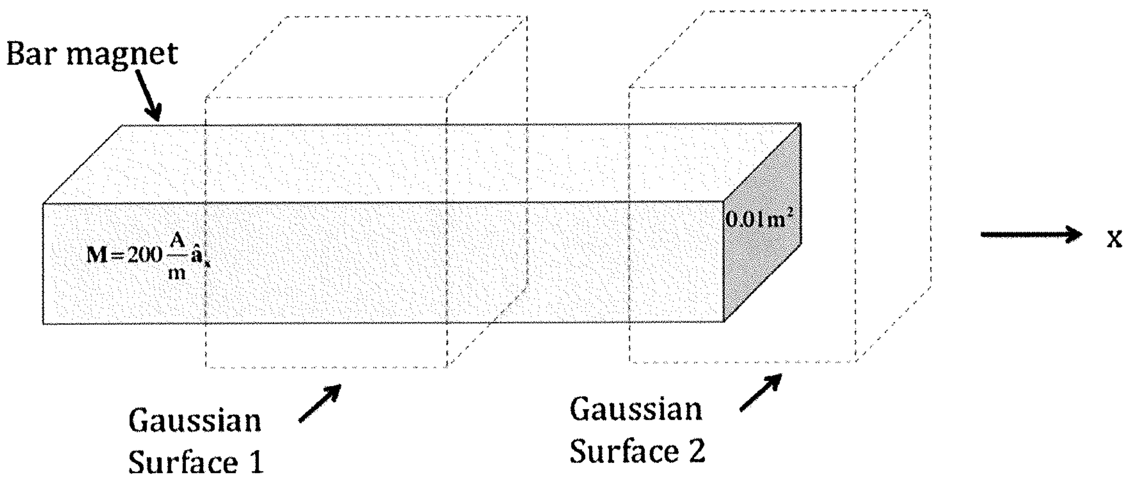
$$\vec{F}_3 = \int_{1m}^{0} (5A)\,dz\,\hat{a}_z \times \frac{2\times 10^{-6}}{\rho} T\hat{a}_\phi$$

$$= \int_{1m}^{0} (5A)\frac{2\times 10^{-6}}{2} T\,dz\,\hat{a}_\rho$$

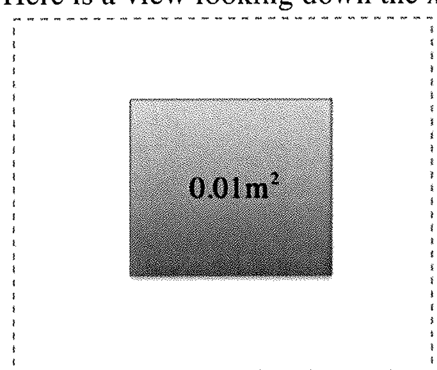
$$= 5\times 10^{-6}\hat{a}_\rho \, N$$

$$F = \vec{F}_1 + \vec{F}_3 = -5\times 10^{-6}\hat{a}_\rho \, N$$

(10 pts) 13. Shown is a bar magnet with magnetization $\mathbf{M} = 200\frac{\text{A}}{\text{m}}\hat{\mathbf{a}}_x$. The right end of the magnet has area 0.01m^2. What is $\oint \mathbf{H} \bullet d\mathbf{S}$ over the two Guassian surfaces shown? You must show your reasoning.



Here is a view looking down the x axis.



$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$\oint \vec{B} \cdot d\vec{s} = 0 = \oint \mu_0 (\vec{H} + \vec{M}) \cdot d\vec{s}$$

$$\oint \vec{H} \cdot d\vec{s} = -\oint \vec{M} \cdot d\vec{s}$$

surface 1

$$\oint_1 \vec{H} \cdot d\vec{s} = -\oint_1 \vec{M} \cdot d\vec{s} = -\left[\left(200\tfrac{A}{m}\hat{a}_x\right) \cdot \left(-0.01m^2 \hat{a}_x\right) + \left(200\tfrac{A}{m}\hat{a}_x\right) \cdot \left(0.01m^2 \hat{a}_x\right)\right]$$

$$= -\left[-2Am + 2Am\right] = 0$$

$$\boxed{\oint_1 \vec{H} \cdot d\vec{s} = 0}$$

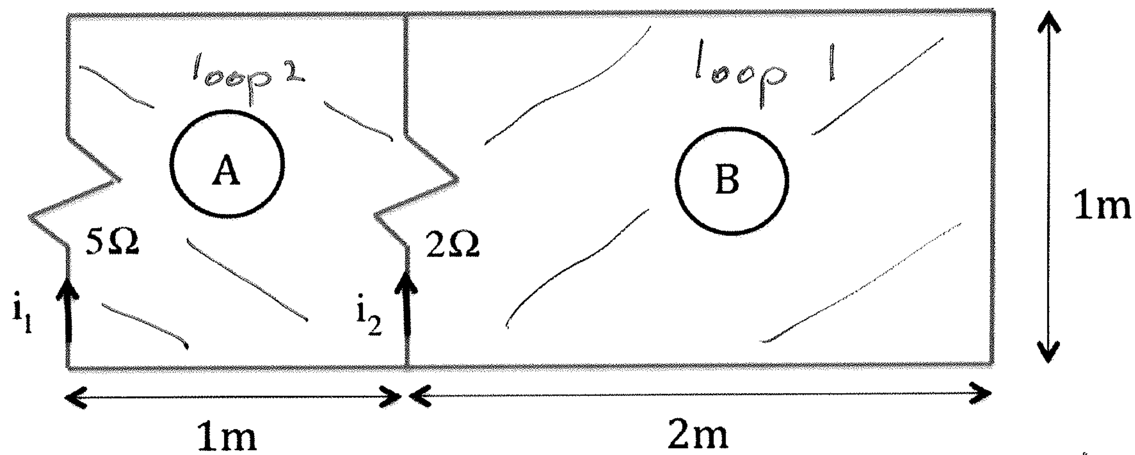
surface 2

$$\oint_2 \vec{H} \cdot d\vec{s} = -\oint_2 \vec{M} \cdot d\vec{s} = -\left[\left(200\tfrac{A}{m}\hat{a}_x\right) \cdot \left(-0.01m^2 \hat{a}_x\right)\right]$$

$$= -(-2Am) = 2Am$$

$$\boxed{\oint_2 H \cdot ds = 2Am}$$

(10 pts) 14. Shown is a circuit in the plane of the page. Through the page are two solenoids. The current is ramped through solenoid A so that $\left(\frac{d\psi}{dt}{}_A\right)_{out-of-page} = 5\text{V}$ and through solenoid B so that $\left(\frac{d\psi}{dt}{}_B\right)_{out-of-page} = 10\text{V}$. What are i_1 and i_2?



loop 1 $\quad \oint_{cw} \vec{E}\cdot\vec{dl} = -\left(\frac{d\psi}{dt}{}_{B \text{ into the page}}\right) = \frac{d\psi}{dt}{}_{B \text{ out-of-the page}}$

$$i_2(2\Omega) + 0 = 10\text{V}$$

$$\boxed{i_2 = 5\text{A}}$$

loop 2 $\quad \oint_{cw} E\cdot dl = -\left(\frac{d\psi}{dt}{}_{A \text{ into the page}}\right) = \frac{d\psi}{dt}{}_{A \text{ out-of-the page}}$

$$i_1(5\Omega) - i_2(2\Omega) = 5\text{V}$$

$$i_1(5\Omega) - (5\text{A})(2\Omega) = i_1(5\Omega) - 10\text{V} = 5\text{V}$$

$$i_1 = \frac{15\text{V}}{5\Omega} = 3\text{A}$$

(10 pts) 15. Write the equation for the electric field for a TEM wave propagating in the direction $-\hat{\mathbf{a}}_z$ in a nonmagnetic dielectric with dielectric constant $\varepsilon_r = 9$, with frequency $f = 3\text{x}10^8$ Hz and amplitude $5\frac{\text{V}}{\text{m}}\hat{\mathbf{a}}_x$. (The speed of an electromagnetic wave in a vacuum is $3x10^8 \frac{m}{s}$ and the intrinsic impedance of free space is 377 Ω.)

Find the velocity of the wave

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_o 9\epsilon_o}} = \frac{1}{3\sqrt{\mu_o\epsilon_o}} = \frac{3\times10^8 \frac{m}{s}}{3} = 1\times10^8 \frac{m}{s}$$

$$\omega = 2\pi f = \left(2\pi \frac{\text{radians}}{\text{cycle}}\right)\left(3\times10^8 \frac{\text{cycles}}{s}\right) = 6\pi\times10^8 \frac{\text{radians}}{s}$$

$$\omega = 6\pi\times10^8 \ s^{-1}$$

find β

$$u = \frac{\omega}{\beta} = \frac{6\pi\times10^8 \ s^{-1}}{\beta} = 10^8 \frac{m}{s}$$

$$\beta = \frac{6\pi\times10^8 \ s^{-1}}{10^8 \ m/s} = 6\pi \ m^{-1}$$

$$\vec{E}(z,t) = 5\cos\left(6\pi\times10^8 t + 6\pi z\right) \frac{V}{m} \hat{a}_x$$

(13 pts) 16. Two non-magnetic lossless dielectrics have their interface at z = 0. For z < 0 the permittivity is ε_o and for z > 0 the permittivity is $4\varepsilon_o$. There is an incident wave in the z < 0 region of $E(z,t) = 9\cos[(6\times10^8 s^{-1})t - (2m^{-1})z]\hat{a}_x \ \frac{V}{m}$. What are the transmitted and reflected electric field intensity waves? (The speed of an electromagnetic wave in a vacuum is $3x10^8 \frac{m}{s}$ and the intrinsic impedance of free space is 377 Ω.)

First determine the intrinsic impedances for η_1 for $z<0$ and η_2 for $z>0$

$$\eta_1 = \sqrt{\frac{\mu_o}{\epsilon_o}} \qquad \eta_2 = \sqrt{\frac{\mu_o}{4\epsilon_o}} = \frac{1}{2}\eta_1$$

Now determine the reflection and transmission coefficients.

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{2}\eta_1 - \eta_1}{\frac{1}{2}\eta_1 + \eta_1} = \frac{-\frac{1}{2}\eta_1}{\frac{3}{2}\eta_1} = -\frac{1}{3}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2(\frac{1}{2}\eta_1)}{\frac{1}{2}\eta_1 + \eta_1} = \frac{\eta_1}{(\frac{3}{2}\eta_1)} = \frac{2}{3}$$

$$E_{ro} = \Gamma E_{io} = (-\tfrac{1}{3})(9\tfrac{V}{m}) = -3\tfrac{V}{m}$$

$$E_{to} = \tau E_{io} = (\tfrac{2}{3})(9\tfrac{V}{m}) = 6\tfrac{V}{m}$$

The frequencies will be the same in the two regions but since the velocities will be different so will the βs, the wave numbers velocity in region 1

$$u_1 = \frac{1}{\sqrt{\mu_o \epsilon_o}} = 3 \times 10^8 \ \frac{m}{s}$$

velocity in region 2

$$u_2 = \frac{1}{\sqrt{\mu_o 4\epsilon_o}} = \frac{1}{2} \frac{1}{\sqrt{\mu_o \epsilon_o}} = 1.5 \times 10^8 \ \frac{m}{s}$$

also $\quad u_2 = \frac{\omega}{\beta_2} \quad$ so $\quad \beta_2 = \frac{\omega}{u_2}$

$$\beta_2 = \frac{6 \times 10^8 \ s^{-1}}{1.5 \times 10^8 \ m/s} = 4 \ m^{-1}$$

We can now write the equations for the reflected and transmitted waves

$$\vec{E}_r(z,t) = -3 \cos\left[6 \times 10^8 t + 2z\right] \hat{a}_x \ \frac{V}{m}$$

$$E_t(z,t) = 6 \cos\left[6 \times 10^8 t - 4z\right] \hat{a}_x \ \frac{V}{m}$$

(10 pts) 17. A TEM wave in free space has the magnetic field intensity

$H(z,t) = 9\cos[(6\times10^8 s^{-1})t - (2m^{-1})z]\hat{a}_x \ \frac{A}{m}$. Find the corresponding electric field intensity.

(The speed of an electromagnetic wave in a vacuum is $3x10^8 \frac{m}{s}$ and the intrinsic impedance of free space is 377 Ω.)

We will need the intrinsic impedance

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377\,\Omega$$

$$E_o = H_o \eta = \left(9\frac{A}{m}\right)(377\,\Omega) = 3{,}393\,\frac{V}{m}$$

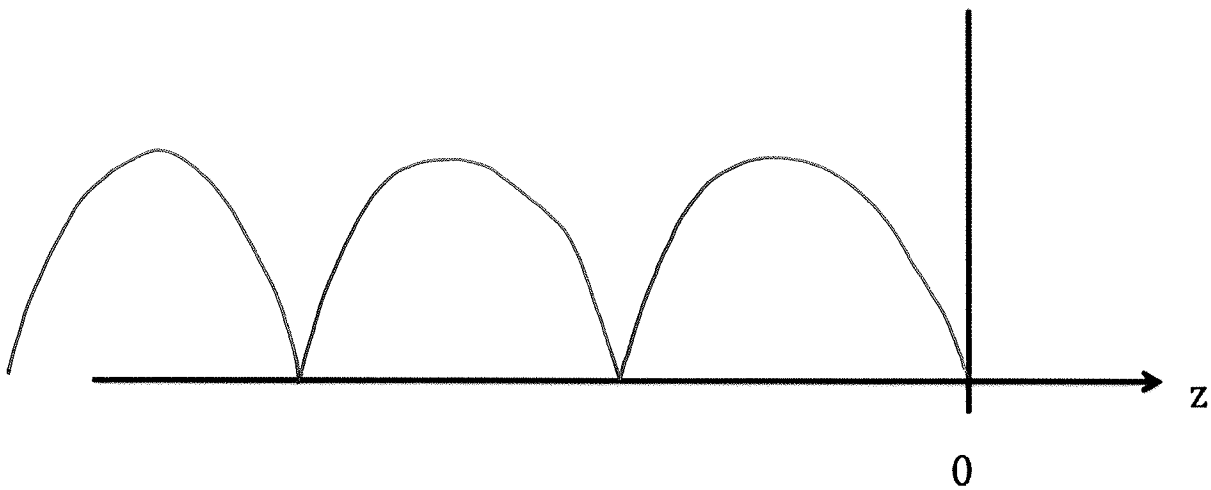
$$\vec{E}(z,t) = -3{,}393\cos[(6\times10^8)t - 2z]\,\hat{a}_y\ \frac{V}{m}$$

note $\vec{\mathcal{P}} = \vec{E}\times\vec{H}$ has to be in the $+\hat{a}_z$ direction and why there is a minus sign

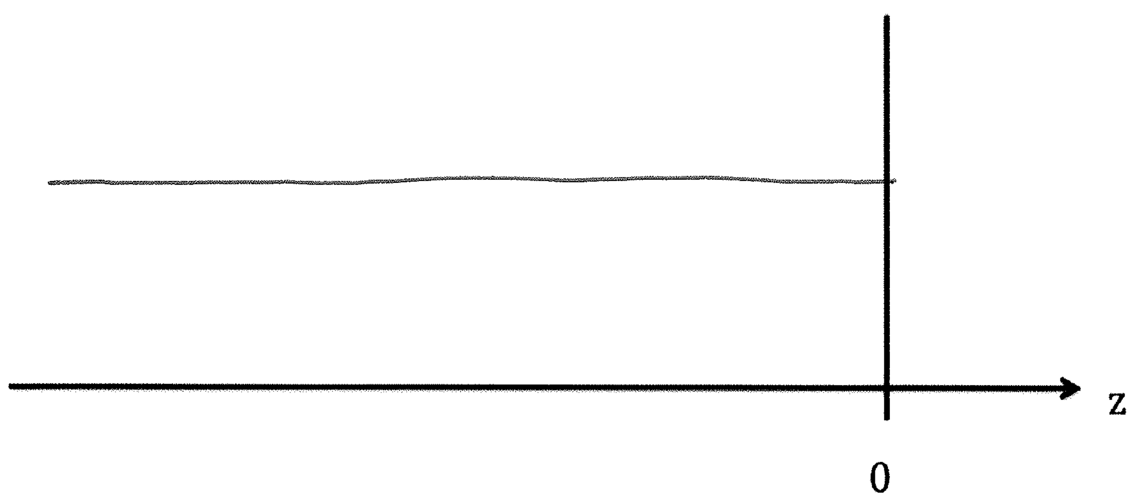
(10 pts) 18. The region for z < 0 is air with permittivity ε_0 Sketch the amplitude of the electric field intensity for z < 0. Choose an amplitude to be consistent with

(The speed of an electromagnetic wave in a vacuum is $3x10^8 \frac{m}{s}$ and the intrinsic impedance of free space is 377 Ω.)

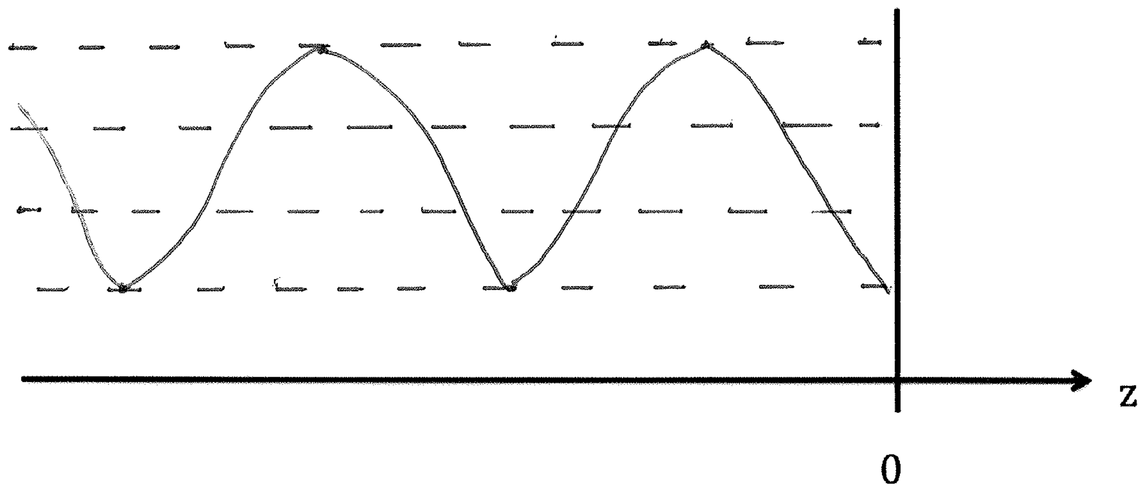
(3 pts) A) If the region for z > 0 is a perfect conductor.



(3 pts) B) If the region for z > 0 has the same dielectric constant as air.



(4 pts) C) If the dielectric constant for the region $z > 0$ is $\varepsilon_r = 16$



$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} \qquad \eta_2 = \sqrt{\frac{\mu_0}{16\,\epsilon_0}} = \frac{1}{4}\eta_1$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{4} - 1}{\frac{1}{4} + 1} = \frac{-\left(\frac{3}{4}\right)}{\left(\frac{5}{4}\right)} = -\frac{3}{5}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{3}{5}}{1 - \frac{3}{5}} = \frac{\left(\frac{8}{5}\right)}{\left(\frac{2}{5}\right)} = 4$$